

Holographic Approach to Regge Trajectory and Rotating D5 brane

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Abstract

We study the Regge trajectories of holographic mesons and baryons by considering rotating strings and D5 brane, which is introduced as the baryon vertex. Our model is based on the type IIB superstring theory with the background of asymptotic $AdS_5 \times S^5$. This background is dual to a confining supersymmetric Yang-Mills theory (SYM) with gauge condensate, $\langle F^2 \rangle$, which determines the tension of the linear potential between the quark and anti-quark. Then the slope of the meson trajectory (α'_M) is given by this condensate as $\alpha'_M = 1/\sqrt{\pi\langle F^2 \rangle}$ at large spin J . This relation is compatible with the other theoretical results and experiments. For the baryon, we show the importance of spinning baryon vertex to obtain a Regge slope compatible with the one of N and Δ series. In both cases, mesons and baryons, the trajectories are shifted to large mass side with the same slope for increasing current quark mass.

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1 Introduction

Among many observations of hadronic phenomena, the Regge behavior, $J = \alpha_0 + \alpha'_{\text{eff}} M^2$, for the hadronic mass M and its spin J has implied the existence of strings which connect constituent quarks. The string theory, however, has been established as the quantum theory of the gravity in ten dimensional (10D) space-time. On the other hand, $SU(3)$ Yang-Mills theory has been established as the basic one to form hadrons in 4D space-time. So the two theories seem to be separated as different one.

After being uncovered these facts, new paradigm in the research of the strong coupling Yang-Mills theory has been opened by the conjecture known as gauge/gravity duality or AdS/CFT correspondence [1, 2, 3]. Many trials to understand strongly interacting gauge theories have been revived within this paradigm. The approach is based on the 10D string theory, which describes gravitational theory. Using the low energy effective action of this theory, many classical solutions have been studied as the dual of the 4D Yang-Mills theories. This approach is also called as the holography since the 4D gauge theory is reflected in the higher dimensional gravity.

Here we study the Regge behavior of hadrons from the gauge/gravity duality. Two possible ways are considered in this approach. One way is to examine the fluctuation modes of probe brane embedded in the supergravity background dual to the confining theory. The approaches in this direction have been performed from five dimensional effective gravity [4, 5, 6]. The string theory is however not yet solved in curved space-time, then it is still an open problem to introduce higher spin states in a strict sense.

Another way is to examine classical string-configurations in the supergravity background dual to the considering Yang-Mills theory. In this case, higher spin meson-states are introduced through the rotation of the strings, whose two end points are on the flavor brane introduced as a probe. While such states are not quantized, we can see the relation of mass and spin for mesons in terms of these solutions at least for large spin. Here we study the Regge behavior through classical solutions as performed in the references [7, 8, 9, 10, 11, 12, 13].

In the $\mathcal{N}=2$ supersymmetric case with flavor brane(s) [7, 11, 12], the background is given by AdS_5 and Regge-like behavior is observed at small J as

$$J = \frac{1}{2\pi m_q^2} \sqrt{\frac{\lambda}{\pi}} M^2, \quad (1.1)$$

where m_q and λ denote the current quark mass and the 'tHooft coupling. In this case, however, the Regge behavior is not observed at large J because the potential between quark and anti-quark is not linear at large distance. The behavior in the region of small J is common to the theory dual to the background which approaches to AdS_5 in the UV limit or near the boundary. Then this behavior corresponds to the behavior of the CFT. Here skipping this CFT behavior at small J , we concentrate on the Regge

trajectory given at large J in the confining theory.

In the background dual to the confining theory, we expect a linear Regge trajectory at large J . In the D4 branes background with flavor D6 probe brane model, such behavior has been observed [9, 10, 11] at the limit of $m_q \rightarrow 0$,

$$J = \alpha' \left(\frac{R_{D4}}{U_h} \right)^{3/2} M^2, \quad (1.2)$$

where α' , R_{D4} and U_h represent the string tension, the radius of S^4 of bulk space-time and the scale of compact S^1 of world volume of D4 brane respectively. It should be noticed that main parameters of the bulk 10D background are remained in this formula. For finite m_q , however, the trajectory deviates from the linear behavior. This deformed behavior has been discussed related to the heavy quarkonium [10]. In any case, as a common property of this kind of classical model, J arrives at zero for $M = 0$, and then the intercept of the trajectory is always zero. In order to have a non-zero intercept, quantum corrections would be necessary as pointed out in [6, 8]. Here we do not extend our analysis to such quantum version. We study the Regge trajectory by using only the classical solutions.

In our approach, the 10D background configuration, which is dual to a confining supersymmetric Yang-Mills theory (SYM), is set as the solution of type IIB supergravity with five form field flux, dilaton and axion [14, 15]. In this model, the tension τ_M of the linear potential between the quark and anti-quark is given as $\tau_M = \sqrt{\langle F^2 \rangle}/2$ by the gauge condensate $\langle F^2 \rangle$ [17]. The mesons and baryons are constructed by embedding probe D7 brane (flavor brane), fundamental strings (F-strings) as quarks and D5 brane as baryon vertex. The D7 brane is introduced to incorporate flavor quarks. The F-strings could end on this D7 brane. These are all introduced as probes, so they do not change the background configuration. That is, the vacuum of the gauge theory is not altered by these branes.

The meson with higher spin is given by a rotating F-string whose both ends on the D7 brane. Its stable configuration is obtained by solving the equation of motion for the Nambu-Goto action of the F-string in the bulk background. In this case, appropriate boundary conditions for strings on the D7 brane are imposed. Since the mass and the spin of the meson are not quantized as mentioned above, then they are continuous. At large J , as expected, we find a linear trajectory,

$$J = \frac{1}{\pi \sqrt{\langle F^2 \rangle}} M^2, \quad (1.3)$$

with the slope parameter $\alpha'_M = 1/(2\pi\tau_M) = 1/\pi\sqrt{\langle F^2 \rangle}$. We should notice that the above formula is expressed only by the quantity of the gauge theory side without any parameter of the bulk side. This result is consistent with the picture obtained from

naive quark model. That is, the rotating quark and anti-quark are connected by a string with the tension of τ_M . On the other hand, at small J , we could find the behavior found in the case of the AdS_5 background mentioned above. The behavior in this region depends on the current quark mass m_q , which is determined by the profile function of the embedded D7 brane.¹ As mentioned above, this meson configuration gives $J = 0$ at $M = 0$, then the intercept is zero.

In the next, we extend this approach to the baryon trajectories, where the baryons are constructed according to the model proposed before [18]-[23]. To make baryons, we must introduce D5 brane as the vertex which attracts N_c F-strings. The F-strings connect the D5 and D7 branes, and then both F-strings and D5 brane contribute to the spin and mass in the case of the baryon. While the D5 vertex could have various configurations, for the simplicity, we examine here two typical configurations given in [23]. One is called as “point vertex”, which does not extend in the real three dimensional space. So it looks like a point in our space. Another one is called as “split vertex”, which extends in one direction in our three space like a string. From the two end point of this vertex, F-strings are going out toward the D7 brane.

In the case of the point vertex, spin comes from the rotating F-strings only since the point vertex couldn't generate rotation. As a result, we find that the Regge slope of this baryon is given by $\alpha'_B = 2\alpha'_M/(N_c(1 + \beta))$ with a positive constant β . In this case, Then it may be difficult to realize the desirable slope parameter in this case for $N_c \geq 3$.

On the other hand, for the case of split vertex, the smaller the energy of the state is, the shorter the attached strings become, then the configuration may be dominated by the vertex. The vertex volume doesn't shrink to zero and remains with a finite size then with a finite energy. This means that the lowest energy state is finite contrary to the mesons. Another point to be noticed is that the split vertex rotates to generate the spin. So we estimate the spin and the mass of the baryon by the vertex only in this case. Then the Regge slope of this configuration is given by the tension of the split vertex. We find a reasonable slope parameter for the case of $N_c = 3$ and 4. It is favorite to consider the case of $N_c = 3$, but a problem to be solved is remained in the case of $N_c = 3$ as discussed below. Here we add the analysis for $N_c = 4$ since this case is free from such a problem.

The outline of this paper is as follows. In the next section, the bulk solution for our holographic model is given. In the section 3, meson trajectories are studied by using Nambu-Goto action. Introducing D5 brane, the baryon trajectories are studied in the section 4. The summary and discussions are given in the final section.

¹This is given by the value of the profile function at UV limit of the fifth coordinate (r). However, in the present supersymmetric case, the profile function is a constant.

2 Holographic Model

Here we give the ingredient of our holographic model for supersymmetric Yang-Mills theory with quarks to form meson and baryons.

2.1 Bulk Background

We consider 10D IIB model retaining the dilaton Φ , axion χ and self-dual five form field strength $F_{(5)}$. Under the Freund-Rubin ansatz for $F_{(5)}$, $F_{\mu_1 \dots \mu_5} = -\sqrt{\Lambda}/2 \epsilon_{\mu_1 \dots \mu_5}$, and for the 10d metric as $M_5 \times S^5$ or $ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{ij} dx^i dx^j$, the solution given below has been found [14, 15]. Where $(\mu, \nu) = 0 \sim 4$ and $(i, j) = 5 \sim 9$. The five dimensional part (M_5) of the solution is obtained by solving the following reduced 5D action,

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left(R + 3\Lambda - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2 \right), \quad (2.1)$$

which is written in the string frame.

The solution is obtained under the ansatz,

$$\chi = -e^{-\Phi} + \chi_0, \quad (2.2)$$

which is necessary to obtain supersymmetric solutions. The solution is expressed as

$$\begin{aligned} ds_{10}^2 &= G_{MN} dX^M dX^N \\ &= e^{\Phi/2} \left\{ \frac{r^2}{R^2} A^2(r) \left(-dt^2 + (dx^i)^2 \right) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right\}. \end{aligned} \quad (2.3)$$

Then, supersymmetric solution is obtained as

$$e^\Phi = 1 + \frac{q}{r^4}, \quad A = 1, \quad (2.4)$$

where $M, N = 0 \sim 9$ and $R = \sqrt{\Lambda}/2 = (4\pi g_s N_c \alpha'^2)^{1/4} = (\lambda \alpha'^2)^{1/4}$ and $\lambda = 4\pi g_s N_c$ denotes the 'tHooft coupling. The dilaton is set as $e^\Phi = 1$ at $r \rightarrow \infty$, and the parameter q corresponds to the vacuum expectation value (VEV) of gauge fields strength [15] of the dual theory. Then this solution is dual to the four dimensional $\mathcal{N}=4$ SYM theory with a constant gauge condensate. Due to this condensate, the supersymmetry is reduced to $\mathcal{N}=2$ and then the conformal invariance is lost since the dilaton is non-trivial as given above. As a result, the theory is in the quark confinement phase since we find a linear rising potential between quark and anti-quark with the tension $\sqrt{q}/(2\pi\alpha'R^2)$ [14, 15, 17].

Furthermore, we can see that the space-time is regular at any point. In the ultraviolet limit, $r \rightarrow \infty$, the dilaton part e^Φ approaches to one and the metric (2.3) is reduced to $AdS_5 \times S^5$. On the other hand, the dilaton part e^Φ diverges in the infrared limit $r \rightarrow 0$,

so that one may expect a singularity at $r = 0$. However there is no such a singular behavior. This is assured by rewriting the metric (2.3) in terms of new coordinate z , where $z = R^2/r$. Then we obtain

$$ds_{10}^2 = e^{\Phi/2} \frac{R^2}{z^2} \left(-dt^2 + (dx^i)^2 + dz^2 + z^2 d\Omega_5^2 \right). \quad (2.5)$$

In the infrared limit $z \rightarrow \infty$, we have

$$e^{\Phi/2} \frac{R^2}{z^2} = R^2 \sqrt{\frac{q}{R^8} + \frac{1}{z^4}} \sim \frac{\sqrt{q}}{R^2}. \quad (2.6)$$

Therefore we find 10D flat space time in this limit and no singular point [14, 15].

2.2 D7 brane embedding

In this background a probe D7 brane is embedded to introduce $\mathcal{N}=2$ hypermultiplet, which corresponds to a string connecting the stacked D3 branes and the probe D7 brane. This is regarded as the quark hereafter. The world volume of the D7 brane is set by rewriting the extra six dimensional part of (2.3) as

$$\begin{aligned} \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 &= \frac{R^2}{r^2} (dr^2 + r^2 d\Omega_5^2) = \frac{R^2}{r^2} \left(\sum_{i=4}^9 dX^{i^2} \right) \\ &= \frac{R^2}{r^2} \left(d\eta^2 + \eta^2 d\Omega_3^2 + \sum_{i=8}^9 dX^{i^2} \right). \end{aligned} \quad (2.7)$$

Then the world volume coordinate of D7 brane is taken as the four dimensional space-time x^μ and extra four dimensions, (η, Ω_3) . Then the induced metric on the D7 brane is expressed as

$$ds_8^2 = e^{\Phi/2} R^2 \left\{ \frac{r^2}{R^2} A^2(r) \left(-dt^2 + (dx^i)^2 \right) + \frac{1}{r^2} \left((1 + w'(\eta)^2) d\eta^2 + \eta^2 d\Omega_3^2 \right) \right\}. \quad (2.8)$$

where $r^2 = \eta^2 + w(\eta)^2$ and $w'(\eta) = \partial_\eta w(\eta)$. Here the D7 brane is embedded under the ansatz,

$$(X^8)^2 + (X^9)^2 = w^2(\eta). \quad (2.9)$$

The solution of $w(\eta)$ is obtained as $w = \text{constant}$ in the background considered here [17]. While, for the simplicity, the solution is given as $(X^8, X^9) = (w, 0)$ in [17], we can choose it at any point of the circle $(X^8)^2 + (X^9)^2 = w$ since the background is symmetric in the (X^8, X^9) plane. Within the probe approximation, $N_f \ll N_c$, N_f number of D7 branes may be put on this circle. We need here at least two D7 branes, $N_f = 2$, to obtain the split vertex given in the section 4. Anyway, for this solution, we can make a simple image for the baryon configuration as shown in the section 4.

Remembering the relation, $z = R^2/r$, the distance between D7 and D3 branes is given by a constant $r_{D7} = R^2/z_{D7}$, then the current quark mass is obtained as

$$m_q = \frac{r_{D7}}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi z_{D7}}. \quad (2.10)$$

2.3 Rotating string

Mesons are given by a open string which two end points are on the D7 brane. The baryons are made by N_c strings, whose one end is on the D7 brane and the other end is on a vertex for every string. The vertex is given by a probe D5 brane which wraps on S^5 of extra dimension[18] in this model. Here the rotating string is formulated as follows.

In the following analysis we change the coordinate r as $R^2/r = z$, then we rewrite the metric (2.3) as follows

$$ds_{10}^2 = e^{\Phi/2} \frac{R^2}{z^2} \left\{ A^2(z) \left(-dt^2 + (dx^i)^2 \right) + dz^2 + z^2 d\Omega_5^2 \right\}. \quad (2.11)$$

Further, the metric for the string which rotates around the x_3 axis is rewritten by cylindrical polar coordinates as,

$$ds_{(5)}^2 = e^{\Phi/2} \frac{R^2}{z^2} \left(A^2(z) \left(-dt^2 + d\rho^2 + \rho^2 d\tilde{\theta}^2 + dx_3^2 \right) + dz^2 \right). \quad (2.12)$$

Taking the string world sheet as $(\tau, \sigma) = (t, z)$ and the ansatz, $\rho = \rho(z)$ and $\tilde{\theta} = \omega t$, the induced metric is given as

$$g_{\tau\tau} = e^{\Phi/2} \frac{R^2}{z^2} A^2(z) \left(-1 + \omega^2 \rho^2 \right), \quad g_{\sigma\sigma} = e^{\Phi/2} \frac{R^2}{z^2} \left(A^2(z) \rho'^2 + 1 \right), \quad (2.13)$$

where prime denotes the derivative with respect to z . Then we have

$$S_{\text{string}} = \int dt \mathcal{L} = -\frac{1}{2\pi\alpha'} \int dt dz e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \sqrt{(1 - \omega^2 \rho^2) (\rho'^2 + A^{-2}(z))}. \quad (2.14)$$

From this, the spin J_s and the energy E_s of this string are given as

$$J_s = \frac{\partial \mathcal{L}}{\partial \omega} = \frac{1}{2\pi\alpha'} \int dz e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \omega \rho^2 \sqrt{\frac{\rho'^2 + A^{-2}(z)}{1 - \omega^2 \rho^2}}, \quad (2.15)$$

$$E_s = \omega \frac{\partial \mathcal{L}}{\partial \omega} - \mathcal{L} = \frac{1}{2\pi\alpha'} \int dz e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \sqrt{\frac{\rho'^2 + A^{-2}(z)}{1 - \omega^2 \rho^2}}. \quad (2.16)$$

These are estimated by giving appropriate solutions for the corresponding strings in mesons and baryons as shown below.

Here we emphasize that the D7 and fundamental strings introduced here are all treated as probes, so they do not alter the bulk background and their embedded configurations. We only impose consistent boundary conditions at their connecting points in constructing the hadron configurations. As for the D5 brane, we explain it in the section 4, where the baryons are discussed.

3 Regge trajectory of Meson

The meson with higher spin is constructed by rotating F-string whose end points are on the D7 brane. The configuration of such a meson state is obtained by solving the equation of motion for the Nambu-Goto action (2.14) with an appropriate boundary condition at the D7 brane. Using this solution, we find the relation of the spin (2.15) and the mass (2.16) to obtain the Regge trajectory.

3.1 Rough Estimation of Slope parameter for small m_q

It is difficult to obtain analytic solution for the string. So before giving the numerical estimations, we give a rough estimation of the relation $J \propto E^2$ in the case of light quark mass. At the small quark mass limit, $m_q \rightarrow 0$, the speed of the quark approaches to the light velocity, $\omega \rho_b \rightarrow 1$, where ρ_b denotes the value of ρ at the end points of the string. They are on the D7 brane. In this limit, the integrations of (2.15) and (2.16) are approximated as follows.

At first, we rewrite (2.15) and (2.16) by changing the integration variable from z to ρ as

$$E_M = 2 \int_0^{\rho_b} d\rho M(\rho), \quad M(\rho) = \frac{1}{2\pi\alpha'} e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \sqrt{\frac{1 + A^{-2}(z)(\partial_\rho z)^2}{1 - \omega^2 \rho^2}}. \quad (3.1)$$

$$J_M = I\omega, \quad I = 2 \int_0^{\rho_b} d\rho \rho^2 M(\rho). \quad (3.2)$$

Here we can regard $M(\rho)$ and I as the mass density and the inertial moment of the rotating string respectively. Since $M(\rho)$ depends on ρ through $z(\rho)$, which is obtained as the solution of the equations of motion, it is hard to perform the integration in (3.1). In the present case, we can however estimate it by a reasonable approximation, which is ensured by the numerical estimations given later.

$M(\rho)$ is written as

$$M(\rho) = \mathcal{M}_{\text{sta}} \sqrt{\frac{1}{1 - \omega^2 \rho^2}}, \quad (3.3)$$

$$\mathcal{M}_{\text{sta}} = \frac{1}{2\pi\alpha'} e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \sqrt{1 + A^{-2}(z)(\partial_\rho z)^2}. \quad (3.4)$$

We notice that M_{sta} is the integrand in estimating the potential of the static quark and the anti-quark. As we have previously shown, we find a linear potential in our present model at large distance between the quark and the anti-quark. This implies, for large ρ_b , that we obtain the following result

$$E_{\text{sta}} = 2 \int_0^{\rho_b} d\rho \mathcal{M}_{\text{sta}} = 2\tau_M \rho_b, \quad (3.5)$$

where the tension, τ_M , is given in our model as follows [17]

$$\tau_M = \frac{\sqrt{q}}{2\pi\alpha' R^2}. \quad (3.6)$$

This result is understood as follows. For large ρ_b , the string shape approaches to the U shape with a long bottom horizontal-line, which is the main part of the string configuration. In the range of this part, $R^2/z = r$ is small and almost constant. Then in this region, we can set as

$$\mathcal{M}_{\text{sta}} \simeq \frac{\sqrt{q}}{2\pi\alpha' R^2} = \tau_M \quad (3.7)$$

for the supersymmetric case (2.4), which is written also as,

$$e^\Phi = 1 + \tilde{q} z^4, \quad \tilde{q} = \frac{q}{R^8}, \quad A = 1. \quad (3.8)$$

Hereafter, we consider this solution for the simplicity. In this approximation, the both side parts of U-shaped string configuration are neglected. They corresponds to the effective quark and anti-quark masses. So the approximation is applied to the case of small quark mass.

Here we notice the following relation [15]

$$q = \pi^2 \langle F^2 \rangle \lambda \alpha'^4, \quad (3.9)$$

where $\lambda = 4\pi g_s N_c$ and $\langle F^2 \rangle$ denote the 'tHooft coupling and the VEV of the gauge field condensate. Then, remembering $R^4 = \lambda \alpha'^2$, we find

$$\tau_M = \frac{1}{2} \sqrt{\langle F^2 \rangle} \quad (3.10)$$

This implies that the tension does not depend on the other parameters, R and α' , and then it is determined only by the mass scale, $\langle F^2 \rangle$, of the confining gauge theory. We notice here another mass scale, R , which determines the meson mass obtained from the D7 brane fluctuations [16]. We also observe the Regge behavior with this mass scale when we restrict the region to the small mass and to the finite quark mass case. In this case, the trajectory is given by (1.1) as mentioned in the Introduction. This behavior is seen as a common properties of the meson spectra through the probe brane fluctuations.

Then \mathcal{M}_{sta} can be approximated by the QCD-string tension τ_M for large J and small quark mass. Under this approximation, E_M and J_M are estimated as follows,

$$E_M = 2\tau_M \int_0^{\rho_b} d\rho \sqrt{\frac{1}{1 - \omega^2 \rho^2}}, \quad I = 2\tau_M \int_0^{\rho_b} d\rho \frac{\rho^2}{\sqrt{1 - \omega^2 \rho^2}}. \quad (3.11)$$

and using the approximation $\omega \rho_b \simeq 1$, we get

$$E_M = \pi \tau_M \rho_b, \quad J_M = \frac{\pi}{2} \tau_M \rho_b^2. \quad (3.12)$$

As a result, we obtain

$$J_M = \frac{1}{2\pi \tau_M} E_M^2. \quad (3.13)$$

This implies that the slope of the meson trajectory is given by

$$\alpha_M^{\text{Regge}} = \frac{1}{2\pi \tau_M} = \frac{1}{\pi \sqrt{\langle F^2 \rangle}}. \quad (3.14)$$

The result (3.13) is interesting since its form is parallel to the one obtained in the flat bulk-space analysis [8], where the Regge slope is obtained as

$$\alpha_M^{\text{Regge}} = \frac{1}{2\pi T_0}, \quad T_0 = \frac{1}{2\pi \alpha'}. \quad (3.15)$$

We find that our result is obtained by replacing the F-string tension T_0 by the QCD-string tension τ_M in the above formula. This point is well understood from the fact that we find 10D Minkowski space-time in the IR limit of our model as shown above. However, we should notice that the scale of the coordinates are redefined as $(q^{1/4}/R)x^\mu \rightarrow x^\mu$.

3.2 Quark mass effect

In the next, we consider the case with a finite m_q , which can not be negligible. We rewrite the lagrangian (2.14) according to the formulation performed in [10] as follows,

$$\mathcal{L} = \int_{z_-}^{z_+} L(z) dz = \int_{z_-}^{z_l} L(z) dz + \int_{z_l}^{z_r} L(z) dz + \int_{z_r}^{z_+} L(z) dz, \quad (3.16)$$

where

$$L(z) = -\frac{1}{2\pi \alpha'} P(z) \sqrt{(1 - \omega^2 \rho^2) (\rho'^2 + 1)}, \quad (3.17)$$

and

$$P(z) = e^{\Phi/2} A^2(z) \frac{R^2}{z^2}. \quad (3.18)$$

The coordinates z_\pm denotes the two end points of the F-string at the D7 flavor brane. The region (i) $z_l < z < z_r$ represents the bottom range of the U-shaped string configuration, which is considered in the above subsection. Then, the two parts, (ii)

$z_- \leq z \leq z_l$ and (iii) $z_r \leq z \leq z_+$, are corresponding the rotating quark and anti-quark respectively. Since the configuration is symmetric, in the present case, then they are equal. And they are neglected in the above as very small quantity. Here we estimate them.

We solve the string equations in terms of the above lagrangian, (3.16). The solution has the U-shape, so we approximate as

$$|\rho'(z)| \gg 1, \quad (3.19)$$

for region (i)

$$\rho'(z) \sim 0, \quad (3.20)$$

for region (ii) and (iii). Then the variation with respect to ρ is given as

$$\begin{aligned} \delta\mathcal{L} = & -\frac{1}{2\pi\alpha'} \int_{z_l}^{z_r} dz \left\{ -\partial_z \left(P(z) \sqrt{1 - \omega^2 \rho^2} \frac{\rho'}{\sqrt{1 + \rho'^2}} \right) - P(z) \sqrt{1 + \rho'^2} \frac{\omega^2 \rho}{\sqrt{1 - \omega^2 \rho^2}} \right\} \delta\rho \\ & - \frac{1}{2\pi\alpha'} \left[P(z) \sqrt{1 - \omega^2 \rho^2} \frac{\rho'}{\sqrt{1 + \rho'^2}} \delta\rho \right]_{z_l}^{z_r} \\ & - \frac{1}{2\pi\alpha'} \left\{ \int_{z_-}^{z_l} dz P(z) \frac{-\omega^2 \rho}{\sqrt{1 - \omega^2 \rho^2}} \delta\rho + \int_{z_r}^{z_+} dz P(z) \frac{-\omega^2 \rho}{\sqrt{1 - \omega^2 \rho^2}} \delta\rho \right\}. \end{aligned} \quad (3.21)$$

The second boundary terms come from the partial integration of the variation for the first term of (3.16). This must be cancelled out with the terms in the third parenthesis of (3.21) since the first term vanishes due to the equation of motion. Noticing the symmetric configuration, we find the next condition,

$$1 - \omega^2 \rho_b^2 = 2\pi\alpha' \frac{\omega^2 \rho_b}{P(z^*)} m_q, \quad (3.22)$$

$$m_q = -\frac{1}{2\pi\alpha'} \int_{z_r}^{z_+} dz P(z), \quad (3.23)$$

where m_q denotes quark mass. Here, further we set as $z_l = z_r = z^*$ and $\rho(z^*) = \rho_b$. The negative sign of the above (3.23) comes from $\rho'(z_r) \sim -\infty$. We notice that the position of the D7 brane is given by $r_f = R^2/z_\pm$. Since $z_+ < z_r$, we have positive m_q . The same relation between ρ_b and m_q is obtained in from the other side condition. This relation is consistent with the fact that the speed of the massless quark is one, namely $\omega\rho_b = 1$ for $m_q = 0$.

Then, under the condition of small m_q but non-zero, we estimate J_M and E_M according to (2.15) and (2.16) by separating the region of z as above. Using the condition given above at the separating point, we obtain

$$E_M = \pi\tau_M(\rho_b + \gamma\rho_b^{1/2}), \quad (3.24)$$

$$J_M = \frac{1}{2}\pi\tau_M \left(\rho_b^2 + 2\gamma\rho_b^{3/2} \right), \quad (3.25)$$

where

$$\gamma = \frac{2}{\pi} \sqrt{\frac{2\pi\alpha' m_q}{P(z^*)}}. \quad (3.26)$$

Then J_M is expressed by E_M by eliminating ρ_b . It is further expanded as a series of γ as

$$J_M = \frac{E_M^2}{2\pi\tau_M} - \frac{\gamma^2}{2\pi^2} E_M + O(\gamma^3). \quad (3.27)$$

The second term is negative and proportional to m_q . Then we would find a shift of the trajectory to the right in the $J - E^2$ plane as the quark mass effect. We should notice, however, that the slope given at large E_M is not changed by m_q .

3.3 Numerical estimations

Equations of motion for SUSY background

The values of these quantities are obtained by introducing the D7 brane to consider quarks with the finite mass. Here, we consider the supersymmetric case, (2.4) or (3.8). In order to solve the string equation, it is convenient to use the reparametrization invariant formalism. The solutions $\rho(z)$ are multi-valued for z , however the configuration of one solution is given by a continuous curve. So the solution can be expressed by one parameter, s as given here. The Lagrangian is written by using the parameter s as

$$\mathcal{L} = -\frac{1}{2\pi\alpha'} \int ds \tilde{L} = -\frac{1}{2\pi\alpha'} \int_{s_l}^{s_r} ds e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \sqrt{(1 - \omega^2 \rho^2) (\dot{\rho}^2 + \dot{z}^2 A^{-2}(z))}. \quad (3.28)$$

where dot denotes the derivative with respect to s . s_l and s_r are defined as

$$z(s_l) = z(s_r) = z_{D7}, \quad \dot{\rho}(s_l) = \dot{\rho}(s_r) = 0. \quad (3.29)$$

The first two equations means that the end points of the string are on the D7 brane, then the second equations are coming from the Dirichlet condition $\rho'(z_{D7}) = 0$ for the string end points on the D7 brane. We impose these boundary conditions in solving $z(s)$ and $\rho(s)$ by using (3.28).

The equations to be solved are obtained as follows. Introducing the canonical momentum as,

$$p_\rho = \frac{\partial \tilde{L}}{\partial \dot{\rho}}, \quad p_z = \frac{\partial \tilde{L}}{\partial \dot{z}}, \quad (3.30)$$

we have the Hamiltonian

$$H = 2 \frac{\tilde{H}}{\Delta}, \quad \Delta = \frac{F}{\sqrt{\dot{\rho}^2 + \dot{z}^2 A^{-2}(z)}}, \quad (3.31)$$

$$F = e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \sqrt{1 - \omega^2 \rho^2}, \quad (3.32)$$

$$\tilde{H} = \frac{1}{2} (p_\rho^2 + p_z^2 A^2(z) - F^2). \quad (3.33)$$

Then the Hamilton equations are obtained from \tilde{H} instead of H for the simplicity,

$$\dot{\rho} = p_\rho, \quad \dot{z} = p_z A^2(z), \quad (3.34)$$

$$\dot{p}_\rho = -\omega^2 \rho Q(z), \quad \dot{p}_z = -p_z^2 A(z) \frac{\partial A(z)}{\partial z} + \frac{1}{2} (1 - \omega^2 \rho^2) \frac{\partial Q(z)}{\partial z}, \quad (3.35)$$

and

$$Q = e^\Phi A^4(z) \frac{R^4}{z^4}. \quad (3.36)$$

In solving the above simultaneous equations (3.34) and (3.35), the boundary conditions (3.29) are imposed as mentioned above. And we need the values of the parameters used in the equations to get numerical results.

About the value of $\lambda = R^4/\alpha'^2$

We expect holographic approaches to be useful at large $\lambda = R^4/\alpha'^2$ and large N_c . Although, in our real world, $N_c = 3$ and this is not large enough, we dare to compare our results with the experiments obtained up to now. We know the ρ meson trajectory, which is approximated by $J = 0.53 + 0.88E^2$. So, from (3.14), we can set as

$$\alpha_M^{\text{Regge}} = \frac{1}{\pi \sqrt{\langle F^2 \rangle}} = 0.88 \text{ (GeV}^{-2}\text{)}. \quad (3.37)$$

This implies

$$\sqrt{\langle F^2 \rangle} = 0.36 \text{ (GeV}^2\text{)}. \quad (3.38)$$

This is our prediction for the gauge condensate. As for the value of this quantity, many attempts to estimate it have been done in 4D gauge theory side. We compare our result with the one given in [24],

$$\langle \frac{g_{\text{YM}}^2 N_c}{4\pi^2} F^2 \rangle = 0.14 \text{ (GeV}^4\text{)}, \quad (3.39)$$

according to the lattice simulation. Here the value is obtained for quenched approximation and for $N_c = 3$. Our calculation is also quenched for the quarks, then they can be compared. Noticing $g_{\text{YM}}^2 N_c = 4\pi g_s N_c = \lambda$, we find

$$\lambda = 42. \quad (3.40)$$

Then λ is not small, thus it may be reasonable to compare our result with the quantities in our real world.

In applying the above result (3.40), we should notice that our model is based on the self-dual gauge field configurations which satisfies $FF = F\tilde{F}$. Then we should study also the value of $F\tilde{F}$ to check the validity of using the result given in the lattice theory, where the supersymmetry is however lost. As in the case of FF , we have searched for simulations of $F\tilde{F}$ in lattice theories, but we could not find any definite result for this quantity. The lattice formulation might be unsuitable to study topological quantities like $F\tilde{F}$.

Even if the self-dual relation for the gauge fields in the background were largely broken in the real QCD, it would be still meaningful to show our analysis since we expect that our main results would survive as being approximately true in the real QCD. Although this point is open, our results are not so sensitive to the value of λ given above as shown below.

Numerical results

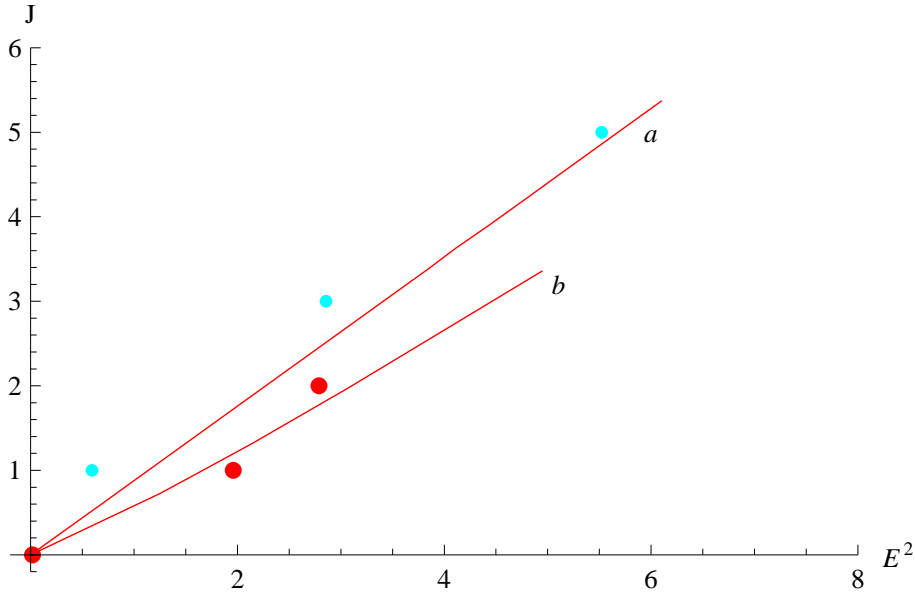


Fig. 1: The numerical result of $J - E^2$ relations for $\tilde{q} = 1/(\cdot 88^2 \times 42) = 0.0306$ and $R^2/\alpha' = 6.48$. The curves (a) and (b) denote the results for $1/z_{D7} = 0.005$, and 0.50 respectively. Here the curves are almost the same for $20 \leq \lambda \leq 42$. And the small (large) dots denote the ρ (π) meson series.

In obtaining the numerical values of J and E , we must fix the position of the D7 brane or the quark mass $\lambda^{1/2}/(2\pi z_{D7}) = m_q$, and other parameters, \tilde{q} (or q) and R^2/α' (or λ). The latter two are fixed by neglecting the quark mass and using our formula, experimental ρ meson trajectory and a result of lattice simulation. The results are given in the previous subsection (3.2).

In Fig.1, two numerical results are shown for $\tilde{q} = 0.0306$ and $R^2/\alpha' = 6.48$ with the mass spectrum of ρ and π meson series. The two curves, (a) and (b) are for $1/z_{D7} = 0.05$ and 0.5 , respectively, where $1/z_{D7}$ denotes the position of the D7 brane and it is equivalent to the quark mass m_q . For very small m_q the trajectory is linear in almost all region and is consistent with (3.14). Further we find the expected shift to the right of the trajectory for the large m_q case, $1/z_{D7} = 0.5$. As for the quark mass dependence, we can see that the heavier the quark mass becomes, the larger the energy is for the same J . This is a reasonable result. The value of λ is set as 42 in the figure, we notice however that the results are the same for wide range of λ , $20 < \lambda < 42$.

We notice one more important point that the slopes of the two curves are different at small mass region. While, for any quark mass, we could find the same slope given by (3.14) at large meson mass as discussed above, we can see the difference of the slope, which depends on the quark mass m_q , in the region of small meson mass. This behavior in small meson mass region is consistent with the trajectory given by (1.1). In this sense, then we could see two regions where the mass scale of the string tension is different.

However we should notice that both of the curves approach to the origin for $J \rightarrow 0$. This implies that the intercept of the trajectory gives zero in the present case. This is understood as follows. In our calculation, the classical solutions of the Nambu-Goto action are used, and they are U-shaped and give positive E . When E vanishes, the string configuration also disappears since the calculation is classical. Then, J vanishes simultaneously for $E = 0$. This implies that the intercept should be zero in this calculation. One way to get finite intercept, in this approach, is to add the quantum effect for the string configurations obtained here. In this case, we could expect to get a positive intercept as given for the closed string case in [8]. We, however, do not consider the quantum corrections in this paper, and we concentrate on the slope parameter of the Regge trajectory at large E and J , where the classical approximation would be good.

4 Baryon Regge trajectory and rotating vertex

The baryon is constructed from the vertex and N_c fundamental strings. The vertex is given by the D5 brane which wraps S^5 of the metric (2.3) (or (2.5)) given above and couples to the 5 form fluxes of the bulk through the $U(1)$ gauge fields in it as shown below. Due to this dissolved fluxes, the baryon vertex could have non-trivial configurations.

In order to show explicit configurations of the baryon vertex considered here, we parametrize the S^5 by the five angle variables, $\{\theta, \theta_2, \theta_3, \theta_4, \theta_5\}$. Then the coordinate $X^i (i = 4 \sim 9)$ of (2.7) are written as,

$$X^9 = r \cos \theta,$$

$$\begin{aligned}
X^8 &= r \sin \theta \cos \theta_2, \\
X^7 &= r \sin \theta \sin \theta_2 \cos \theta_3, \\
X^6 &= r \sin \theta \sin \theta_2 \sin \theta_3 \cos \theta_4, \\
X^5 &= r \sin \theta \sin \theta_2 \sin \theta_3 \sin \theta_4 \cos \theta_5, \\
X^4 &= r \sin \theta \sin \theta_2 \sin \theta_3 \sin \theta_4 \sin \theta_5,
\end{aligned} \tag{4.1}$$

and we have $\eta^2 = \sum_{i=4}^7 X^i{}^2 = r^2 \sin^2 \theta \sin^2 \theta_2$.

In general, we can consider many kinds of configurations for the D5 brane. They will be shown by introducing the external coordinates on the D5 brane as, $r(\theta_i)$ and $X^i(\theta_j)$, $i = 1 \sim 3$, but it would be very complicated to solve the equations of motion to minimize the action for these general functions. Then, we restricted our analysis to a simple case with $r(\theta)$ and $(X^1(\theta))^2 + (X^2(\theta))^2 \equiv \rho^2(\theta)$. In this sense, our configurations adopted here are not special but simple. For more complicated configurations, they are remained as future works.

The two typical configurations of baryons considered here are shown in the Figs.2 and 3 in the three dimensional space of $\{X^9, X^8, \eta\}$, where the distance from the origin is equivalent to the coordinate r , which is denoted as $r = \sqrt{(X^9)^2 + (X^8)^2 + \eta^2}$.

As mentioned above, the D5 brane is embedded to minimize its action with the coordinates $\{r(\theta), \rho(\theta)\}$, where $\rho^2 = (X^1)^2 + (X^2)^2$. Unfortunately, in the Figs.2 and 3, we can not see the coordinate ρ and then the extension of $\rho(\theta)$ for the split vertex. The point and split vertices are discriminated here by the number of the cusp, namely one for the point and two for the split as shown in the Figs. 2 and 3.

As for the D7 brane, its configuration is expressed by the solution given in (2.9) in the form of a tube of $w = \text{constant}$. We should notice here about the number of the D7 branes. In the Fig. 2, the point vertex is shown with one cusp of D5 brane. In this case, we need one D7 brane at $(X^8, X^9) = (0, w)$ as the end point of strings as shown in the figure. On the other hand, we need at least two flavor D7 branes at $(X^8, X^9) = (0, \pm w)$ for the split vertex baryon as shown in the Fig. 3. This implies that the number of the flavor quarks might be related to the configuration of the baryon vertex, especially to the number of the cusps on the D5 brane. This point should be examined more by considering more complicated configurations in the future.

As mentioned above, both D branes are embedded as probes. Then we neglect the back reactions. However, we should notice that there are cusps in D5 brane due to the dissolved bulk flux as $U(1)$ fields in it. On the other hand, the fundamental strings, which terminate on these cusps, are dissolved in the D5 brane [18, 19] as the $U(1)$ flux. Then there is a smooth flow of the strings from D5 brane vertex.

On the other hand, the embedded D7 brane has no cusp, and then strings end on this brane with the Diriclet conditions. This situation is the same with the case of mesons, which are given as strings whose two end-points are on the D7 branes with the same boundary condition. For the case of the baryon, however, the situation would be

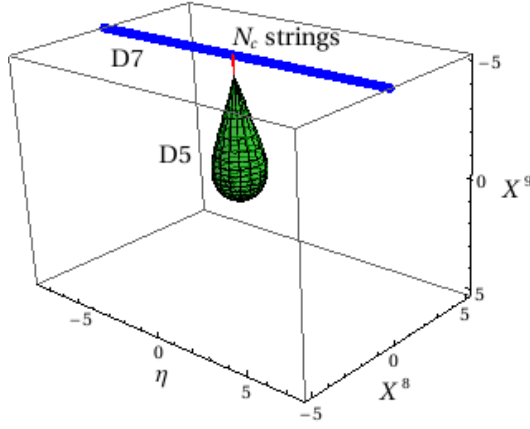


Fig. 2: The typical configuration of point vertex(D5 brane) and flavor brane(D7 brane) in the space (X^8, X^9, η) . The point vertex has one cusp at $\theta = \pi$, and then all N_c strings should attach to this cusp.

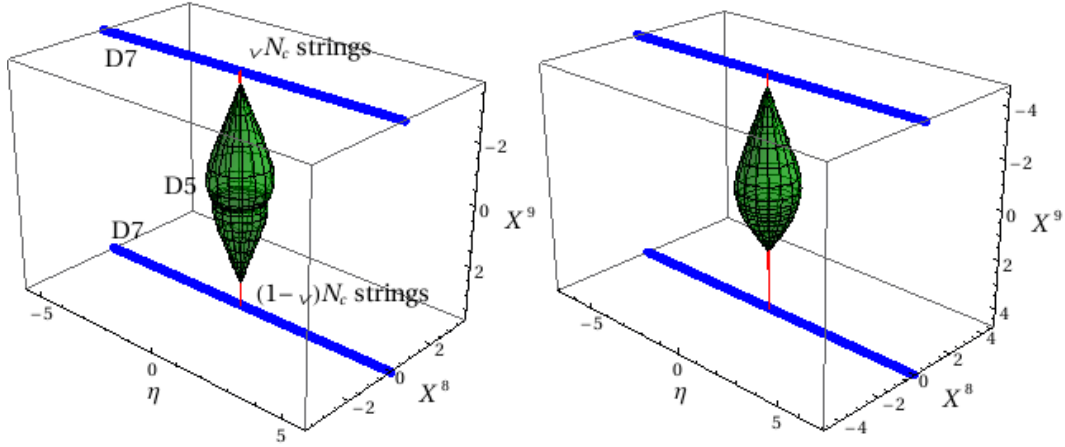


Fig. 3: The typical configurations of split vertex and two flavor branes on (X^8, X^9, η) . The split vertex has two cusp at $\theta = 0, \pi$. νN_c strings should attach to the cusp at $\theta = 0$, and $N_c(1 - \nu)$ strings should attach the other cusp. Two shapes of the vertex are shown. They depend on the boundary conditions of embedding equations, the left one with a spindle-shape and the right with the peanuts form in the figure.

slightly different since the baryon number is not zero due to the dissolved strings. As a result, the $U(1)$ gauge field in the D7 brane would be a non-trivial configuration which reflects the non-zero charge density. This effect however could be neglected since the density of baryon number in the D7 brane would be negligibly small.

In the next, we consider the vertex part, which is given by probe D5 brane.

4.1 Rotating D5 brane vertex

The D5-brane action is thus written by the Dirac-Born-Infeld (DBI) plus Chern-Simons term

$$\begin{aligned} S_{D5} &= -T_5 \int d^6 \xi e^{-\Phi} \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})} + T_5 \int (2\pi\alpha' F_{(2)} \wedge c_{(4)})_{0\dots 5}, \quad (4.2) \\ g_{ab} &\equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}, \quad C_{a_1\dots a_4} \equiv \partial_{a_1} X^{\mu_1} \dots \partial_{a_4} X^{\mu_4} C_{\mu_1\dots\mu_4}. \end{aligned}$$

where $T_5 = 1/(g_s(2\pi)^5 l_s^6)$ and $C_{(4)}$ are the brane tension and induced four form respectively. The Born-Infeld term involves the induced metric g and the $U(1)$ worldvolume field strength $F_{(2)} = dA_{(1)}$. The second term is the Wess-Zumino coupling of the worldvolume gauge field, and it is also written as

$$S = -T_5 \int d^6 \xi e^{-\Phi} \sqrt{-\det(g + F)} + T_5 \int A_{(1)} \wedge G_{(5)},$$

in terms of (the pullback of) the background five-form field strength $G_{(5)} = dC_{(4)}$, which effectively endows the fivebrane with a $U(1)$ charge proportional to the \mathbf{S}^5 solid angle that it spans.

Then we give the embedded configuration of the D5 brane in the given 10D background by minimizing the action. At first, we fix its world volume as $\xi^a = (t, \theta, \theta_2, \dots, \theta_5)$. For simplicity we restrict our attention to the metric (2.12) with the ansatz, $\rho = \rho(\theta)$ and $\tilde{\theta} = \omega t$. Next we consider the $SO(5)$ symmetric configurations of the form $\rho(\theta)$, $z(\theta)$, and $A_t(\theta)$ (with all other fields set to zero), where θ is the polar angle in spherical coordinates. The action then simplifies to

$$S = T_5 \Omega_4 R^4 \int dt d\theta \sin^4 \theta \left\{ -\sqrt{e^\Phi A^2 \left(\frac{R}{z}\right)^4 (1 - \rho^2 \omega^2) (z^2 + z'^2 + A^2 \rho'^2) - F_{\theta t}^2} + 4A_t \right\}, \quad (4.3)$$

where $\Omega_4 = 8\pi^2/3$ is the volume of the unit four-sphere.

The gauge field equation of motion following from this action leads

$$\partial_\theta D = -4 \sin^4 \theta,$$

where the dimensionless displacement is defined as the variation of the action with respect to $E = F_{t\theta}$, namely $D = \delta \tilde{S} / \delta F_{t\theta}$ and $\tilde{S} = S / T_5 \Omega_4 R^4$. The solution to this equation is

$$D \equiv D(\nu, \theta) = \frac{3}{2}(\nu\pi - \theta) + \frac{3}{2} \sin \theta \cos \theta + \sin^3 \theta \cos \theta. \quad (4.4)$$

Here, the integration constant ν is expressed as $0 \leq \nu = k/N_c \leq 1$, where k denotes the number of strings emerging from one of the pole of the S^5 , which is wrapped by the D5 brane vertex. Next, it is convenient to eliminate the gauge field in favor of D and Legendre transform the original Lagrangian to obtain an energy functional of the embedding coordinate only:

$$U_{D5} = \frac{N}{3\pi^2\alpha'} \int dz \frac{R^2}{z^2} e^{\Phi/2} A(z) \sqrt{(1 - \rho^2 \omega^2) \left(1 + z^2 \left(\frac{\partial \theta}{\partial z} \right)^2 + A^2 \left(\frac{\partial \rho}{\partial z} \right)^2 \right)} \sqrt{V_\nu(\theta)} . \quad (4.5)$$

$$V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8 \theta , \quad (4.6)$$

where we used $T_5 \Omega_4 R^4 = N_c / (3\pi^2 \alpha')$, and the integral variable is changed from θ to z . So θ is a function of z in this case. Using this expression (4.5) and (4.6), we can solve the spinning D5 brane configurations.

In the following, we consider two cusps at $\theta = 0$ and $\theta = \pi$, where the displacement is finite and actually we have

$$D(\nu, 0) = \frac{3}{2} \nu \pi , \quad D(\nu, \pi) = \frac{3}{2} (\nu - 1) \pi . \quad (4.7)$$

They should be cancelled by the fundamental strings to form a baryon. We use the above setting of the D5 brane for two typical baryon states. (a) One is set as $\nu = 0$ or 1 and $\rho = 0$. We call this state as point vertex baryon. The D5 brane is seen as a point in our real three space since it does not extend in the direction of ρ . (b) Another configuration is obtained for $0 < \nu < 1$ and $\partial \rho / \partial z \neq 0$. In this case, the D5 brane is extended as a string in the direction of ρ , and the quarks are attached on both side of this extended vertex. The quarks are separated to $N_c \nu$ and $N_c(1 - \nu)$ quarks on the end points of the vertex. This is called as the split vertex baryon. The configurations of these two types of baryonic states are shown in the Figs. 2 and 3. In the next, we examine the spin and the energy of these configurations.

4.2 Spin and Energy for Baryon

(a) Point vertex;

In this case, the D5 brane vertex contributes only to the energy and not to the spin J , which is obtained from the strings attached to the D5 brane point-vertex. Then we have

$$J_P = N_c J_s , \quad E_P = N_c E_s + U_{D5}(\rho = 0)|_{\nu=0} , \quad (4.8)$$

where J_s , E_s and U_{D5} are given by (2.15), (2.16) and (4.5). The configuration of D5 brane for $\nu = 1$ is the same as the one of $\nu = 0$ case, so it is sufficient to consider only the above case. J_P and E_P are calculated as follows. At first, solve the equation of

motion for $z(\theta)$ by using $U_{D5}(\rho=0)|_{\nu=0}$. Then we find the tension of the D5 brane at $\theta = \pi$, and this is balanced by the string part, and this is expressed by the “no-force condition” [23]. This condition provides the boundary condition for the equations of motion of strings given by (2.14). It is obtained at $\rho = 0$ and $\theta = \pi$ as,

$$\left(\frac{\partial z}{\partial \rho}\right)_{\text{string}} = \left(\frac{1}{z} \frac{\partial z}{\partial \theta}\right)_{D5}. \quad (4.9)$$

This condition gives the balance of the force in the z direction at the connecting point of the strings and the D5 brane. The condition for the force in the ρ direction is satisfied by putting the strings symmetrically around the D5 brane vertex as shown in the Fig.4.

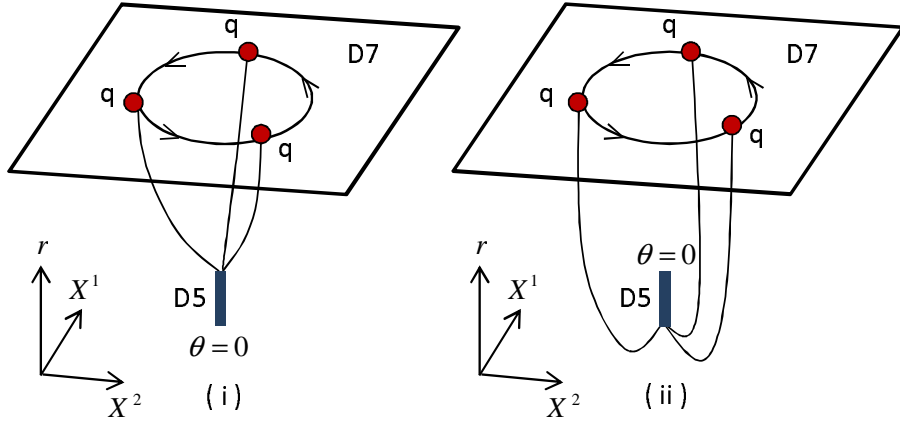


Fig. 4: Rotating baryon configurations for point vertex.

As shown in [23], we find two typical string configurations in the case of the point vertex. They are discriminated by the conditions, (i) $1/z(0) < 1/z(\pi)$ or (ii) $1/z(0) > 1/z(\pi)$ of the D5 brane configuration. They are shown in the Fig.4.

For the case of (i), the strings can not stretch so much, then this configuration contribute only to the small J_P and small E_P baryons. In this case, the lowest energy of the baryon is obtained when the strings shrink up to the D7 brane, and then the baryon is almost constructed by the D5 vertex. This configuration gives the lowest baryon mass, which is finite contrary to the lowest zero meson mass. Here we give the lower bound for this lowest baryon mass. It is obtained from the D5 brane which doesn't stretch to z -direction ($\partial z / \partial \theta = 0$). This is realized when the position of the D5 brane $z = z_{D5}$ is located at the same point of the D7 brane $z = z_{D7}$, namely $z_{D5} = z_{D7}$. Such a solution is given [23] as follows,

$$z_{D5} = z_{D7} = \tilde{q}^{-\frac{1}{4}}. \quad (4.10)$$

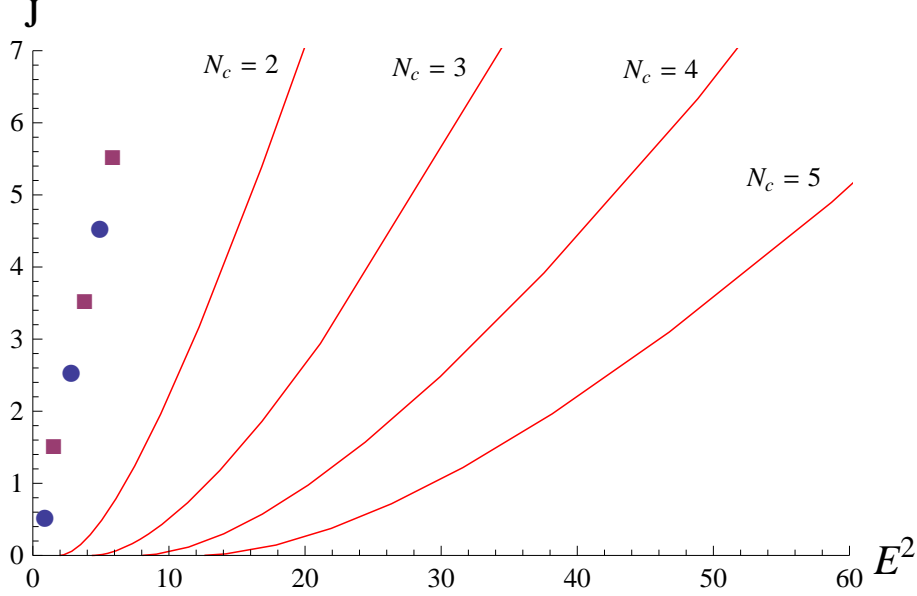


Fig. 5: The baryon trajectories with point vertex are shown for $N_c = 2 \sim 5$. The curves are given for $\alpha'_M = 0.88$ (GeV^{-2}), $m_q = 0.3$ (GeV), and $\lambda = R^2/\alpha' = 10$. The squares and dots denote the Δ and N baryon series respectively.

For the above constant-point solution, we have

$$U_{D5} = \frac{\sqrt{2}N_c}{3\pi^2} \alpha'^{-1/2} \lambda^{1/4} j(0), \quad j(\nu) = \int_0^\pi d\theta \sqrt{V_\nu(\theta)}, \quad (4.11)$$

where $j(0) = 7.6$. When we use the experimental relation, $\alpha'_M = 0.88$ (GeV^{-2}) we find

$$U_{D5} = 0.387 \times N_c \lambda^{1/4}. \quad (4.12)$$

In general, $z_{D5} > z_{D7}$, then for $\lambda > 1$, we have the lowest mass bound, which exceeds the nucleon mass for $N_c = 3$.

For large J_P and E_P , the baryons are expressed by the configuration (ii), we can approximate E_P as

$$E_P \simeq N_c E_s \simeq N_c \frac{(1+\beta)}{2} E_M, \quad (4.13)$$

since the vertex part $U_{D5}(\rho=0)|_{\nu=0}$ is negligible compared to the string part which is stretched long. The second equality is obtained since one of the N_c strings can be replaced by the half and some extra (β) part of the meson (see (ii) of the Fig.4). Then we find following linear relation between J_P and E_P^2 ,

$$J_P \simeq \frac{2}{N_c(1+\beta)} \frac{1}{2\pi\tau_M} E_P^2. \quad (4.14)$$

Then the slope of the linear trajectory at large E_P is given as

$$\alpha_B^{Regge} = \frac{2}{N_c(1+\beta)} \alpha_M^{Regge}. \quad (4.15)$$

This is smaller than $2/N_c \times \alpha_M^{Regge}$, which is expected from naive quark model. According to our numerical analysis, it is about 60 percent of the expected one for $N_c = 3$ here. The reason of this suppression can be reduced to factor $1/(1+\beta)$ in equation (4.14). In the present case, we find $\beta \sim 1/8$ in our numerical calculation given here, but this factor will depends on the configurations of the rotating strings.

On the other hand, the experimental result given by the nucleon series implies $\alpha_B^{Regge} \simeq 0.99 \text{ (GeV}^{-2}\text{)}$. This can not be obtained from the above result (4.15). Then we should consider another baryon configuration for higher spin states.

(b) Split vertex;

In the next, we consider the split vertex, and the baryon is expressed almost by the vertex only. In other words, the strings are absorbed into the vertex and their end points are attached on the two D7 branes. So the baryon configuration considered here is described by the D5 brane, see Fig. 4. Here the two cusps of D5 brane are considered as the strings with zero length, then we impose the boundary condition for the cusps,

$$\partial_z \rho|_{z=z_{D7}} = 0. \quad (4.16)$$

From (4.5) the spin is obtained as

$$J_{D5} = -\frac{\partial U}{\partial \omega} = \frac{N_c}{3\pi^2 \alpha'} \int dz \frac{R^2}{z^2} e^{\Phi/2} A \rho^2 \omega \sqrt{\frac{1 + z^2 \left(\frac{\partial \theta}{\partial z}\right)^2 + A^2 \left(\frac{\partial \rho}{\partial z}\right)^2}{1 - \rho^2 \omega^2}} \sqrt{V_\nu(\theta)}. \quad (4.17)$$

We remember that (4.5) originally includes the $d\tilde{\theta}/dt = \omega$, so it is further Legendre transformed with respect to U . Then we obtain the energy

$$E_{D5} = -\frac{\partial U}{\partial \omega} \omega + U = \frac{N_c}{3\pi^2 \alpha'} \int dz \frac{R^2}{z^2} e^{\Phi/2} A \sqrt{\frac{1 + z^2 \left(\frac{\partial \theta}{\partial z}\right)^2 + A^2 \left(\frac{\partial \rho}{\partial z}\right)^2}{1 - \rho^2 \omega^2}} \sqrt{V_\nu(\theta)}. \quad (4.18)$$

The energy and the spin are evaluated as in the case of the meson. Firstly, we rewrite the above equation (4.18) as

$$E_{D5} = \int_{\rho_L}^{\rho_R} d\rho M_{\text{sta}}^{D5} \sqrt{\frac{1}{1 - \omega^2 \rho^2}}, \quad (4.19)$$

$$M_{\text{sta}}^{D5} = \frac{N_c}{3\pi^2 \alpha'} e^{\Phi/2} A^2(z) \frac{R^2}{z^2} \sqrt{1 + z^2 (\partial_\rho \theta)^2 + A^{-2}(z) (\partial_\rho z)^2} \sqrt{V_\nu(\theta)}, \quad (4.20)$$

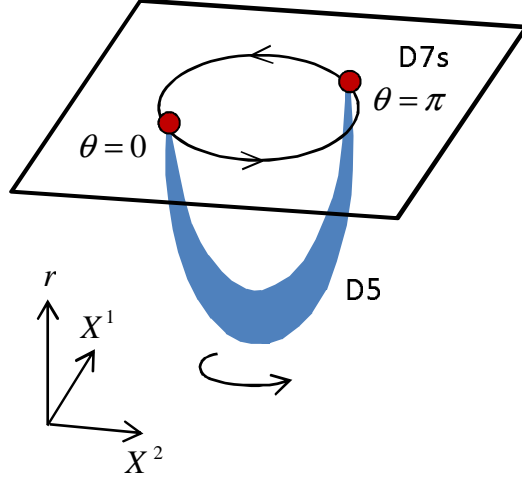


Fig. 6: A typical rotating baryon configuration for split vertex with minimum energy is shown in the 3D space of (X^1, X^2, r) . In this configuration, we can ignore F-strings which connect the D5 and D7 branes since their lengths are almost zero. Therefore F-strings do not contribute to the energy and also to the spin of the baryon. Here we notice that the two D7 branes at different X^9 are sitting at the same r , and the two end points of D5 brane are moving on the different D7 branes.

where ρ_L and ρ_R denote the end points of the D5 brane vertex. For the case of mesons, the end points of the string are given as $\rho_R = -\rho_L = \rho_b$ since the configuration is symmetric. But in the present case, the D5 brane vertex is not symmetric except for $\nu = 1/2$. In spite of this fact, we find a similar relation of energy and spin.

For the case of large E_{D5} , M_{sta}^{D5} can be approximated by the value at $\theta = \theta_c$, where $V_\nu(\theta)$ is maximum. And it is given as the solution of the following equation [23],

$$\pi\nu = \theta_c - \frac{1}{2} \sin(2\theta_c). \quad (4.21)$$

Further, we find

$$\partial_\rho z = \partial_\rho \theta = 0 \quad (4.22)$$

at the point of $\theta = \theta_c$. As a result, we obtain

$$M_{\text{sta}}^{D5} = \frac{2N_c}{3\pi} \sin^3(\theta_c) \tau_M \equiv \tau_B, \quad (4.23)$$

where τ_M is given above Eq. (3.6). Then

$$E_{D5} \simeq \tau_B \int_{\rho_L}^{\rho_R} d\rho \sqrt{\frac{1}{1 - \omega^2 \rho^2}} \simeq \tau_B \pi \rho_R, \quad (4.24)$$

where we used $\omega \rho_R = 1$ and $\omega \rho_L = -1$. J_{D5} is similarly obtained as

$$J_{D5} \simeq \frac{\pi}{2} \tau_B \rho_R^2 = \frac{1}{2\pi \tau_B} E_{D5}^2. \quad (4.25)$$

Then we could obtain the similar result to the meson trajectory with the different slope,

$$\alpha'_B = \frac{1}{2\pi\tau_B}. \quad (4.26)$$

The experimental data of α'_B lead to a similar value with that of meson, $\alpha'_B \simeq \alpha'_M \simeq 1$ ($1/\text{GeV}^2$). Then we expect

$$\frac{2N_c}{3\pi} \sin^3(\theta_c) \simeq 1. \quad (4.27)$$

For $N_c = 2, 3$, we can not obtain a configuration for the split baryon which satisfies the no force condition. The reason is that one string must couple to one of the cusps of D5 brane, and this situation is not allowed due to the no force condition [25]. Then, we find $\frac{2N_c}{3\pi} \sin^3(\theta_c) = 0.85, 1.02$ for $(N_c, \nu) = (4, 1/2), (5, 2/5)$ respectively. In these cases, we can set no force condition and satisfactory slope parameter. These are assured by the numerical calculation given below with numerical exact solutions.

Numerical estimation

In order to obtain the spin and energy, we solve the equations of motion of the action (4.5). It is convenient to rewrite U in the parametrization invariant form to solve the equations.

$$U = \frac{N_c}{3\pi^2\alpha'} \int ds \frac{R^2}{z^2} e^{\Phi/2} A \sqrt{(1 - \rho^2\omega^2) (\dot{z}^2 + z^2\dot{\theta}^2 + A^2\dot{\rho}^2)} \sqrt{V_\nu(\theta)}. \quad (4.28)$$

where dot denotes the derivative with respect to the introduced parameter s , for example $\dot{z} = \partial z / \partial s$. We rewrite U as

$$U = \frac{N_c}{3\pi^2\alpha'} \int ds \tilde{U}, \quad (4.29)$$

where

$$\tilde{U} = Q \sqrt{\dot{z}^2 + z^2\dot{\theta}^2 + A^2\dot{\rho}^2}, \quad Q = \frac{R^2}{z^2} e^{\Phi/2} A \sqrt{(1 - \rho^2\omega^2) V_\nu(\theta)}. \quad (4.30)$$

Then the canonical momenta of $q_i (= (z, \theta, \rho))$ are given by $p_i = \frac{\partial \tilde{U}}{\partial \dot{q}_i}$ as

$$p_z = \frac{\dot{z}}{\Delta}, \quad p_\theta = z^2 \frac{\dot{\theta}}{\Delta}, \quad p_\rho = A^2 \frac{\dot{\rho}}{\Delta}, \quad (4.31)$$

where

$$\Delta = \frac{\sqrt{\dot{z}^2 + z^2\dot{\theta}^2 + A^2\dot{\rho}^2}}{Q}. \quad (4.32)$$

Then the Hamiltonian H is written as

$$H = \sum p_i \dot{q}_i - \tilde{U} = \Delta \left(p_z^2 + \frac{p_\theta^2}{z^2} + \frac{p_\rho^2}{A^2} - Q^2 \right). \quad (4.33)$$

Due to the reparametrization invariance of the system, we have the constraint $H = 0$. This constraint can be changed to a more convenient form by changing the Hamiltonian as

$$H = 2\tilde{H}\Delta, \quad \tilde{H} = \frac{1}{2} \left(p_z^2 + \frac{p_\theta^2}{z^2} + \frac{p_\rho^2}{A^2} - Q^2 \right), \quad (4.34)$$

and the equations of motion are obtained for new Hamiltonian \tilde{H} as

$$\dot{z} = p_z, \quad \dot{\theta} = \frac{p_\theta}{z^2}, \quad \dot{\rho} = \frac{p_\rho}{A^2}, \quad (4.35)$$

$$\dot{p}_z = -\frac{\partial \tilde{H}}{\partial z}, \quad \dot{p}_\theta = -\frac{\partial \tilde{H}}{\partial \theta}, \quad \dot{p}_\rho = -\frac{\partial \tilde{H}}{\partial \rho}. \quad (4.36)$$

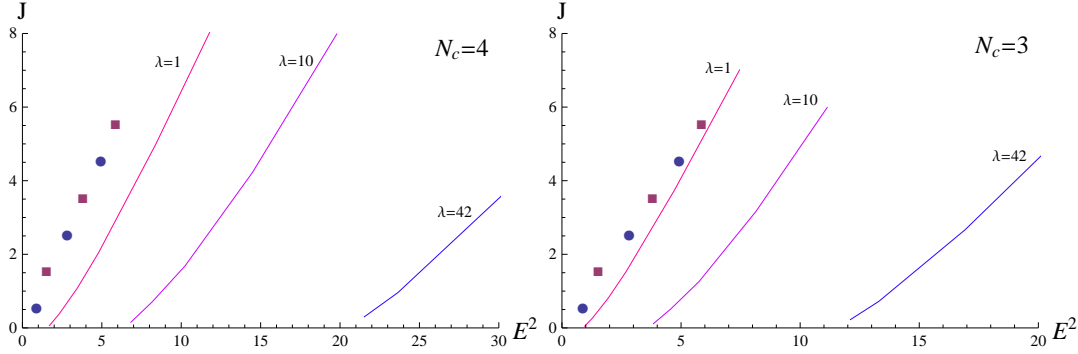


Fig. 7: The baryon trajectories with rotating D5 split vertex are shown for $N_c = 4$ (left) and $N_c = 3$ (right). The curves are given for $\alpha'_M = 0.88$ (GeV^{-2}), $1/z_{D7} = 1.0$ (GeV), and three values of $\lambda = R^2/\alpha'$. The large and small dots denote the Δ and N baryon series respectively.

In Fig. 7, we show our numerical results of the split baryon trajectories for the cases of $\{N_c = 4, \nu = 1/2\}$ and $\{N_c = 3, \nu = 1/3\}$. In each case, the calculations are given for $m_q (= \sqrt{\lambda}/(2\pi z_{D7})) = 0.16, 0.51, 1.04$ (GeV). The slope parameter is compatible with the experimental spectra. However, in the case of baryon, the masses become very large for large λ due to the vertex energy. The vertex does not disappear when the distance between two ends of split vertex becomes zero, and then it remains to give the lowest mass of the baryon. While we expect that this lowest mass would be near the nucleon mass, it depends on λ as shown in the Fig. 7. Since the holographic approach is trusted for $\lambda \gg 1$, it seems to be difficult to get a lowest mass comparable to the nucleon mass. It may be about three or four times of the nucleon mass for $\lambda \sim 5 > 1$. We should notice that the meson trajectory is not changed by the parameter λ as shown above.

Another problem is that we can not get the no-force condition for the split baryon of $N_c = 3$ [25]. The three F-strings going out from the two pole-points of split vertex

are separated to one and two in the case of $N_c = 3$. We impose the no-force conditions on both sides where the F-string(s) and D5 brane couple. However, we know that on the side of one F-string it is impossible to balance the force due to the tension of the one F-string and the one of the D5 brane since the former one is always greater than the latter. As a result, the string shrinks up to zero length and disappears. Then this cusp of the D5 vertex can not be in a stable state. So we should extend to $N_c = 4$ to have a stable split vertex. This is the reason why we show the example of numerical simulation only for $N_c = 4$ at the same time.

One way to allow the baryon state of $N_c = 3$ and $\nu = 1/3$ is to introduce a pair of quark and anti-quark at the cusp with one F-string coupled. Then the no force conditions are satisfied in this case, but there appears another possible instability to annihilate the introduced a pair of quark and anti-quark. The energy to make this vertex state is however very small since the cusp is nearly at the flavor brane position, so we can consider this state within an energy fluctuation of quantum corrections which would be important at this cup point. In this sense, it is meaningful to consider the split vertex also for $N_c = 3$

As a result, we could say that the higher spin baryon states are realized by rotating vertex, which extends as a string with a definite tension. The F-strings, which are attached to the vertex, shrink to almost zero length and don't contribute to the spin of the baryon state. This picture of baryon configuration is different from the one considered by the naive quark model. We could find that the main ingredient of the baryon is not the F-string but the vertex, see Fig. 6.

5 Summary and Discussion

Using a holographic model for mesons and baryons with higher spins, Regge behaviors are studied. In our model, confinement is realized due to the gauge condensate $\langle F^2 \rangle$ which determines the tension τ_M of the linear potential between the quark and anti-quark as $\tau_M = \sqrt{\langle F^2 \rangle}/2$. As a result, the slope of the meson trajectory is given by this quantity as $\alpha'_M = 1/(\pi\sqrt{\langle F^2 \rangle})$, and then $\langle F^2 \rangle$ is fixed from the experimental data of meson spectrum. We obtain $\langle F^2 \rangle = 0.13 \text{ (GeV}^4\text{)}$ from the experimental data of ρ meson trajectory. On the other hand, the intercept of the trajectory is always zero in our model since the meson is approximated by a classical configuration of Nambu-Goto string, which is embedded in a bulk dual to the confining gauge theory. In this model, the string is reduced to a point when its energy approaches to zero. Then the angular momentum also tends to zero. This would be improved by considering the quantum effects, which are not considered here.

As for the baryons, the Regge behavior is examined for two typical configurations, which are discriminated by the vertex. The baryon vertex is given by D5 brane which is introduced in the bulk background as a probe brane with $U(1)$ flux. It wraps on S^5

in the bulk. We consider two simple vertex configurations. One is given by a solution being extended to the fifth coordinate r as $r(\theta)$. In this case, the vertex is observed as a point in our 4D space-time, so we call this as a point vertex. Another configuration is obtained as a solution of the equations of motion of two coordinates, $r(\theta)$ and $x(\theta)$. This configuration extends like a string in the plane of $r - x$, where x is one of our three space coordinates. This is called as split vertex since the vertex is observed as a line extended in the x direction. (See also Figs.2 and 3) We should notice that we need at least two flavor D7 branes for the split vertex baryon as shown in the Fig. 3. This implies that the number of the flavor quarks might be related to the number of the cusps on the D5 brane baryon vertex. We should examine this point more in the future.

In the case of the point vertex (see Fig. 4), spin and energy are mainly given by the strings. Then the slope of the baryon, α'_B , is related to the meson slope as in the form $\alpha'_B = 2\alpha'_M/(N_c(1 + \beta))$, where β denotes a positive factor which depends on the string configuration. Then, in general, $\alpha'_B < \alpha'_M$ for $N_c \geq 3$. As a result, we could not obtain the slope α'_B , which is comparable with the experimental data.

On the other hand, for the split vertex case, the spin and the energy are supposed to be provided by the vertex only since the energy is minimized for this configuration (see Fig. 6). Then we can neglect the contribution from the strings in the analysis. The vertex with higher spin, in this case, extends in one spatial direction x of our three dimensional space. Then the situation is similar to the case of mesons. That is to say that the slope parameter is determined by the tension of this string-like vertex when it extends in the x direction. This tension is in general smaller than the summation of the independent N_c strings, then the slope of the trajectory becomes larger than that of the point-vertex case. Actually we could find a slope, which is compatible with the experimental data, for $N_c = 3$. In this case, the problem is that we must adjust the parameters to obtain the lowest mass of the baryon compatible with the nucleon mass. This is performed by using unfavorably small λ . Furthermore, in order to satisfy the no-force condition at the vertex, we should extend N_c to a large value $N_c \geq 4$. However, this extension moves the baryon mass to the large side. These points should be resolved by including quantum corrections. Another possible improvement will be found in other holographic model of different bulk background and probe branes. They are remained as future works.

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